Aprendizaje Reforzado Profundo para la Administración de Portafolios de Renta Fija

Deep Reinforcement Learning for Automated Fixed Income Portfolio Management

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**Resumen**

En este trabajo se aplican técnicas de aprendizaje reforzado profundo en la administración de portafolios de inversión de renta fija, específicamente títulos soberanos emitidos por el gobierno colombiano. El periodo de análisis comprende siete años, desde enero de 2015 hasta diciembre de 2022. Encontramos que es posible generar rentabilidad y lograr una eficiente gestión del riesgo como resultado de las estrategias de “trading” que los modelos de aprendizaje reforzado profundo prevén más convenientes dadas ciertas condiciones de mercado y de cada uno de los títulos, como su riesgo implícito en métricas como DV01, Duración y Convexidad. Finalmente, este estudio contribuye al muy estudiado campo de las aplicaciones de aprendizaje de máquina e inteligencia artificial sobre predicción del mercado de valores y administración de carteras de inversión.

**Abstract**

This paper applies deep reinforced learning techniques to the management of fixed income investment portfolios, specifically sovereign securities issued by the Colombian government. The period of analysis covers seven years, from January 2015 to December 2022. We find that it is possible to generate profitability and achieve efficient risk management as a result of the trading strategies that deep reinforced learning models foresee more convenient given certain market conditions and of each of the securities, such as their implied risk in metrics like DV01, Duration and Convexity. Finally, this study contributes to the much-studied field of machine learning and artificial intelligence applications on stock market prediction and portfolio management.

*Palabras Clave:* Yield curve; Machine Learning; Trading strategy; Deep Reinforcement Learning; Fixed Income; Risk Management; Portfolio Management.

***JEL Classification:* C610; G190**

# INTRODUCTION

This paper studies how machine learning (ML) can be applied for riding the yield curve strategy in the Colombian curve case. Therefore, the objective is to provide a way of generating additional return for investors applying this strategy and a support for the liquidity preference theory.

Though the bond investment strategies are not brand-new tactics, they are not only gaining interest recently in financial industry but also in academia (see e.g., Jansen, 2020; Lee, 2021). Investors make profits when yields on long-term bonds are higher than on bonds of shorter maturities by employing the riding the yield curve. The latter was the case when short-term rates where nearly zero and longer-term rates were higher.[[1]](#footnote-1) The strategy seems to be profitable when there is an expectation of an important macroeconomic decision from the central banks. For instance, just after the global financial crisis (June 2009), the European Central Bank (ECB) allocated EUR 442 billion at a cost below than the market interest rate allowing 1,121 participant banks earn an additional yield of 350 basic points by riding the yield curve.[[2]](#footnote-2) Another example was in 2015, when the Federal Reserve was thinking to raise rates in the middle of the year. According to Bank of America, employing the strategy on the curve, would produce $4.7 billion of net interest income with a one-percentage-point "parallel shift" in the curve.[[3]](#footnote-3) Thus, different monetary policies provide interest rate patterns and “understanding how the yield curve moves and reacts to different economic events provides a glimpse into the relationships between long- and short-term interest rates. For example, short-term interest rates might be quite low, but that does not guarantee low long-term rates: a steeply sloped yield curve will have long rates much higher than short rates.” (Haubrich, 2004). Even in a negative-yield scenario, as in 2019, hedge funds such as GAM Systematic’s Cantab Quantitative fund, Lynx Asset Management fund, among others could earn profits implementing bond investment strategies.[[4]](#footnote-4)

As a consequence, a trader may use the ‘riding the yield curve’ strategy to obtain profits by anticipating variations in the interest rate. For instance, when the yield curve is upward sloping, an active bond portfolio manager may buy bonds with maturity longer than his investment horizon.

On the other hand, there are three main hypotheses in the financial theory to explain the shape and behavior of the yield curve term structure. These are the pure expectations theory (Fisher, 1896; Kane and Malkiel, 1967), the liquidity preference theory (Hicks, 1939), and the market segmentation hypothesis (Culbertson,1957). For the pure expectations theory, the expected short-term interest rates are mainly explained by the shape of the term structure. In the liquidity preference theory, the yield premium is another important variable that determines the term structure and this premium is proportional with the maturity of the analyzed fixed-income instrument. Under the market segmentation hypothesis (MSH), funds supply and demand determine the interest rate within different maturities of the yield curve. A variation of the MSH is the so-called preferred habitat theory (Modigliani and Sutch, 1966), where the maturity sectors are invested depending on its liabilities. These postulates are analyzed by portfolio managers and investors to earn profits with some trading strategies, such as the riding the yield curve.

In relation to these theories, if the pure expectations theory holds, the riding the yield curve strategy would not generate additional returns (Grieves and Marcus, 1992). On the contrary, our empirical results show that the profits from the strategy are consistent with the liquidity preference theory and are in line with the results found in Pelaez (1997). Thus, we consider that the main contribution of our paper to the literature is the application of machine learning to the riding the yield curve strategy with a unique database from emerging markets, and it is one of the first studies in this field of finance. Another contribution is the potential use of the applied machine learning techniques to design a “slope surprise.” The latter is defined as the unexpected change in the slope of the yield curve following each Federal Open Market Committee (FOMC) announcement (English et al., 2018).

This paper is divided as follows. Section 2 presents the related literature to the seminal empirical studies regarding the yield curve, the rolling down the yield curve strategy, and the main machine learning works applied to the interest rate field. Section 3 shows the methodology employed in our paper, starting with a toy example. Section 4 presents the empirical results of applying the methodology to the Colombian government bond dataset. Finally, Section 5 concludes.

**2. Related Literature**

**2.1. Yield curve studies**

In one of the seminal studies of empirical works about the yield curve, [Hamburger and Platt (](#_bookmark18)1975[)](#_bookmark18) study the short term of the U.S. Treasury bill curve. The authors find that the three-month forward rate for the 1960s is a poor predictor of the future long-term spot rate. On the other hand, F[ama (](#_bookmark14)1984[)](#_bookmark14) verifies the hypothesis of pure expectations and finds that the forward rate is not an unbiased estimator of the future spot rate, therefore, rejecting the hypothesis. Later, F[ama (](#_bookmark15)2006[)](#_bookmark15) finds evidence of the predictive capacity of the forward rate over the future spot rate for horizons greater than one year in the U.S. Treasury bond market for the 1952 – 2004 period. [Campbell and Shiller](#_bookmark11) ([1991](#_bookmark11)) argue that the yield spread does not predict long-term interest rate movements in an accurate manner. Many authors have used different types of models and variables to explain the behavior of the future spot rates, e.g., [Dewachter et al. (](#_bookmark13)2014[)](#_bookmark13) employs macro-finance models to estimate the term premium dynamics. In this study we find that long-term bonds are affected by all macro shocks, in particular with long-run inflation shocks, and that movements in the term premium are associated with financial shocks. Y[un (](#_bookmark25)2019[)](#_bookmark25) investigates the liquidity premium for the Korean government bonds and finds that, similar to the U.S. bond market, the liquidity premium is affected by domestic expected inflation, but additionally, it is also affected by global liquidity factors such as S&P 500 option-implied volatility, bank capital flows, and the leverage of global banks. Furthermore, [Baele et al. (2010)](#_bookmark10) used a dynamic factor model, where stock and bond returns depend on economic state variables where additional to interest rate, inflation, output growth and cash flow growth, the model includes macro-economic uncertainty measures derived from survey data on inflation and GDP growth.

In the Colombian case, [Rey (](#_bookmark23)2[005)](#_bookmark23) found evidence against the hypothesis of pure expectations, using the TES[[5]](#footnote-5) (national government debt bonds) market data between 2000 and 2004. This theory is rejected for all periods except for 80 and 270 days. In another related study, [Rueda and Arango (](#_bookmark24)2008[)](#_bookmark24) find evidence in favor of the liquidity preference theory, based on [Fama](#_bookmark15)’s ([2006](#_bookmark15)) model. The authors also find that the forward rate contains information on the future trends of the spot rate given that for all terms without exception the forward rate is positive and statistically significant.

**2.2. Riding the yield curve**

We examine the application of machine learning algorithms with trading strategies on interest rates. Such strategies are very well-known in practices such as ‘riding the yield curve’. As Martellini et al. (2003) mention “Riding the yield curve is a technique that fixed-income portfolio managers traditionally use in order to enhance returns.” Several works have attempted to verify the efficacy of the strategy. For instance, Dyl and Joehnk (1981) find that this strategy yields returns higher than short-term rates by employing US T-bill from 1970 to 1975. On the other hand, Chandy and Hsueh (1995) revealed that riding the yield curve is not profitable in the US market from 1981 to 1985. Moreover, Grieves and Marcus (1992) show that riding the yield curve stochastically dominates[[6]](#footnote-6) the buy-and-hold strategy during most of the 1979-1988 period, and similar results are found in Grieves et al. (1999). Other studies that find that the riding the yield curve strategy is more profitable than the buy-and-hold strategy are Pelaez (1997), Brieri and Chincarini (2005), Mercer et al. (2009), Whereas Ang et al. (1998) and Chua et al. (2005) find mixed results. From a mean-variance point of view, Galvani and Landon (2012) conclude that the strategy is ineffective only when involving long-term bond portfolios.

**2.3. Machine learning in interest rates**

Though there has been an increasing interest in machine learning applications in finance recently, few studies have been devoted to interest rate modelling. A first attempt of using machine learning techniques in interest rates is performed by Kanevski et al. (2010). The authors employ unsupervised (self-organizing Kohonen maps, Gaussian Mixtures) and supervised machine (multilayer perceptron) learning algorithms to model the Swiss franc interest rate curves. Then, Gogas et al. (2015) employ support vector machine (SVM) to classify recession forecasting by employing the U.S. yield curve.

However, since the salient work of Kondratyev (2018), practitioners and academics are eagerly investigating traditional and new machine learning algorithms to better forecast the term structure of interest rates. In his work, the author employs artificial neural networks (ANN) to examine the behavior of the term structure curve dynamics. The results show that ANN outperforms the most commonly-used tool in practice, the principal component analysis (PCA) on the validation dataset. Similar results are obtained by Chataigner et al. (2020), on at-the-money swaption surfaces, where PCA is not applicable to original data. Nunes et al. (2019) compare multilayer perceptron and linear regression to forecast the European yield curve. The authors find that the MLP with all relevant variables present the best results.

In a related work, Spears et al. (2021) use deep learning to model price change prediction of the Eurodollar Futures curve. Meanwhile, Kim (2021) uses several machine learning techniques (linear regression, multilayer perceptron, support vector machines, random forest, AdaBoost, and Gradient Boosting) to predict the spread of interest rates between two bonds with maturities of 3 and 10 years for the Korean treasury bonds. The results show that the AdaBoost algorithm outperformed other forecasting models. Bianchi et al. (2021) also examine several machine learning techniques (simple and penalized linear regressions, regression trees, random forest, neural networks) compared to the PCA as a benchmark to predict bond excess returns with accurate results according to the out-of-sample forecast indicators. A recent example of machine learning, more specifically deep learning, in mathematical finance is the work of Benth et al. (2021), where the authors employ deep learning to calibrate the Heath-Jarrow-Morton (HJM) model in order to price European-style options obtaining accurate results.

Despite the good results concerning machine learning in the interest rate field shown in the literature, there is still more work to do in this arena. In fact, to the best of our knowledge, there are no works examining machine learning algorithms applied to trading strategies with the yield curve. Our work is intended to complement the already published outstanding works reviewed in this section.

**3. Models and Methodology**

If the liquidity preference theory holds, the expected value of the spot interest rate for a term , negotiated in , will be less than the forward rate negotiated in time for the same periods of time. That is

. (1)

For the Colombian case, [Rey (](#_bookmark23)2005[)](#_bookmark23) and Rueda [and Arango (](#_bookmark24)2008[)](#_bookmark24) found evidence in favor of the liquidity preference theory in this market. This means that the yield curve will have a positive slope and the forward rate is not an unbiased estimate of the future real interest rate, and therefore, under the liquidity preference theory it is possible to execute arbitrage strategies (see e.g., Ang, 1998; Cox & Felton, 1994; Galvani & Landon, 2013; Grieves & Marcus, 1992; Mercer et al., 2009; Pantalone & Platt, 1994; Pelaez, 1997). A simple strategy is to trade a short position at , a long position at , then sell the long position at , and finally close the short position in the period .[[7]](#footnote-7) Therefore, the profit and loss (P&L) of the long position will be represented by , and the short position by . Thus, the final P&L of the strategy will be given by the result of the following expression.

. (2)

Since

, (3)

the equation (2) can be rewritten as

. (4)

By simplifying Equation (4), and assuming that the interest rates are continuous compounding, the P&L in terms of relative change can be expressed as

. (5)

Using the latter equation, we calculate the relative change of the profit and losses, or simply the returns, of the strategy for the periods {90,90; 180,180; 360,360; 720,360; 1080,360; 1440,360; 1800,360; 2160,360; 2520,360; 2880,360; 3240,360}, for the analyzed 04/28/2006 - 02/22/2019 period. For instance, if a strategy is executed for the periods {1080,360}, assuming , a short position is traded today at 1080 days with rate *y*0*,*1080, a long position with *y*0*,*2160, then sell the long position with *y*1080*,*360, and close the short position at time *t* = 1080. Table 1 shows the main descriptive statistics for the strategy returns, which is expressed in annual continuous compounding.

### 

### Table 1. Main descriptive statistics of strategy returns

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 90 - 90 | 180 - 180 | 360 - 360 | 720 – 360 | 1080 - 360 | 1440 - 360 | 1800 - 360 | 2160 - 360 | 2520 - 360 | 2880 - 360 | 3240 - 360 |
| Min | -2.76% | -2.45% | -2.19% | -1.13% | -0.85% | -0.14% | -9.62% | -5.75% | 0.07% | -0.04% | -0.02% |
| Max | 3.44% | 4.95% | 9.35% | 7.63% | 7.69% | 7.21% | 4.97% | 9.93% | 9.42% | 3.25% | 4.28% |
| Average | 0.08% | 0.45% | 1.30% | 1.48% | 1.94% | 1.75% | 0.76% | 0.07% | 2.47% | 1.17% | 1.18% |
| P10 | -0.72% | -0.73% | -0.78% | -0.29% | 0.59% | 0.42% | -0.18% | -1.40% | 1.55% | 0.76% | 0.56% |
| P20 | -0.43% | -0.28% | -0.13% | 0.07% | 1.00% | 1.04% | -0.06% | -1.00% | 1.75% | 0.94% | 0.65% |
| P30 | -0.21% | -0.08% | 0.40% | 0.57% | 1.15% | 1.33% | 0.03% | -0.52% | 1.89% | 1.03% | 0.81% |
| P40 | -0.06% | 0.12% | 0.70% | 0.97% | 1.52% | 1.55% | 0.17% | -0.33% | 2.06% | 1.11% | 0.94% |
| P50 | 0.07% | 0.35% | 1.03% | 1.35% | 1.97% | 1.73% | 0.50% | -0.17% | 2.24% | 1.17% | 1.06% |
| P60 | 0.18% | 0.55% | 1.29% | 1.62% | 2.42% | 1.89% | 0.94% | 0.12% | 2.51% | 1.23% | 1.27% |
| P70 | 0.31% | 0.74% | 1.62% | 1.94% | 2.71% | 2.15% | 1.24% | 0.77% | 2.86% | 1.29% | 1.42% |
| P80 | 0.45% | 1.02% | 2.27% | 2.42% | 2.94% | 2.53% | 1.58% | 1.31% | 3.30% | 1.37% | 1.54% |
| P90 | 0.75% | 1.45% | 3.61% | 3.73% | 3.33% | 2.92% | 2.10% | 1.69% | 3.65% | 1.55% | 1.95% |
| N | 3343 | 3278 | 3147 | 2886 | 2625 | 2363 | 2103 | 1843 | 1582 | 1320 | 1059 |

Min = minimum, Max = maximum, P means percentile (e.g., P10 is the 10th percentile), and N is the count of the strategies in each period. The number of datapoints (N) change depending of the period of the strategy. For instance, the period {360, 360} uses the data from a specific date to a year later, while period {90,90} uses the data 90 days later. The difference between the number of datapoints between a year and a quarter, corresponding to period {360, 360} and {90, 90} is 196 (=3346−3150). Thus, as increases, the number of variables available for prediction decreases.

As observed in Table 1, the median (i.e., the 50th percentile) of the strategy returns is positive for all periods (except the period {2160, 360}), but the average return is positive for all cases. This means that the robust central tendency measure of the returns is positive for most of the periods. It is worth mentioning that for the {2520, 360} period, it is the only case where the minimum return is positive (0.07%). In addition, the second highest maximum (9.42%) is presented in this period, whereas the highest maximum (9.93%) is exhibited in the {2160, 360} period.

# Prediction of the returns for each strategy

In this paper, we use different forecast approaches to predict the profitability of the proposed strategies. In the first place, we apply supervised regression methods. We use classics models for time series such as AR and ARIMA and after that we employ nonparametric Machine Learning methods such as random forest algorithms to compare the model assessment of these models, considering the nature of the data of interest. Subsequently, we implement classification algorithms to expand the performance of the proposed models. Finally, we compare the model assessment of all models by the means of various metrics of goodness of fit and the Diebold-Mariano test.

* + 1. **Data description and exploration**

The data source is the spot rates of Colombian government bonds daily between 26/7/2006 to 22/2/20019. It is important to note that we also explored the contribution of other variables such as implicit inflation, market variables such as the stock market (Colcap index), and the COP-USD exchange rate. All these variables are available on the Bloomberg platform.

**Gráfico, Gráfico de líneas

Descripción generada automáticamenteFigure 1.** Yield Curves’ shapes for the period of analysis

To analyze and describe the statistical and economic properties of the yield curves, we plotted them for different points in time after cleaning and pre-processing the dataset.

Figures 1 and 2 show the pattern in time followed by the curves from 2006 until 2018, as well as the yields per maturity and the yield spreads to three months, respectively. From these figures, we can conclude that the sovereign yield curves present a decline in level for the time of analysis, as well as a gain steepness, especially for the case of the curves of 2012 and 2014. In addition, it is important to note that the yields of maturities of 5 years are higher than the rest consistently, during the whole period, and the yields for the maturities of 7 years were lower. Finally, regarding the yield spreads it can be observed a correlation between the periods of higher spread with the international and national financial crises of 2008, 2012, and 2016.

**Figure 2.** Yields per Maturity and yield spreads to 3M

Interfaz de usuario gráfica, Gráfico, Histograma

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To complement our analysis of the drivers of each point on the curve, we include a graphic of the volatility over 60 days. Accordingly, from the last figure, it can be observed that the periods of more volatility correspond to the international crises of 2008, 2014, and 2016.

**Figure** Interfaz de usuario gráfica, Gráfico, Histograma

Descripción generada automáticamente**3.** Rolling volatility of the yield curves

**Reduction of dimensionality by applying Principal Component Analysis (PCA)**

Considering the objective of prediction to maximize the profit of the trading strategies explained in the methodology section, we decide to reduce the dimensionality of our variables by employing principal component analysis techniques. This type of dimensionality reduction contributes to filtering and reducing data's noise, consequently improving the model’s performance.

On the other hand, as suggested by the literature, applying the yield curve decomposition into its main drivers, we could also delve into the “underlying dynamics” of the yield curve under analysis.

To implement the PCA analysis, we construct our characteristics matrix X with features corresponding to the daily yields for every given maturity. Then, we derived the eigenvectors from the covariance matrix of X by minimizing the distances generated by the projections onto the vector itself. This process guarantees that we can capture the maximum variability of all maturities. Since we are only interested in variance, we centered each of the variables in X to have a mean zero.

Even though the process of the eigenvalue decomposition obtains the same number of vectors as the initial characteristic matrix, we only retained the three most important ones, as these represent 99.72% of the variance explained. As a consequence of this significant quantity of explained variance, the yield curve movements can be approximated by linear combinations of the first three loadings with small relative error, as we will demonstrate in the following sections.

Next, we present in figure 4 the contribution in terms of percentage to the explained variance of the eigenvectors obtained by the PCA process, and in figure 5 the patrons followed by the first three eigenvectors through the maturities. According to the economic meaning proposed by the literature, the first component represents the shifts of the yield curve (level), the second component constitutes tilting of the yield curve (slope), and the third component acts for the curvature of the yield curve.

**Figure 4.** Percentage ofvariance explained by the Eigenvalues

Gráfico

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**Figure 5.** Behavior of the three principal components obtained by PCA

Gráfico, Gráfico de líneas

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Lastly, regarding the interpretability of the PCA results, we plot the Eigen-scores, which also can be compared to the traditional factors: “level”, “slope” and “curvature”. The higher scores for the first component are those associated with years 2006, 2007, 2008, and 2009, and the higher scores for the second component are those with relatively recent years such as 2015, 2014, 2013, and 2016.

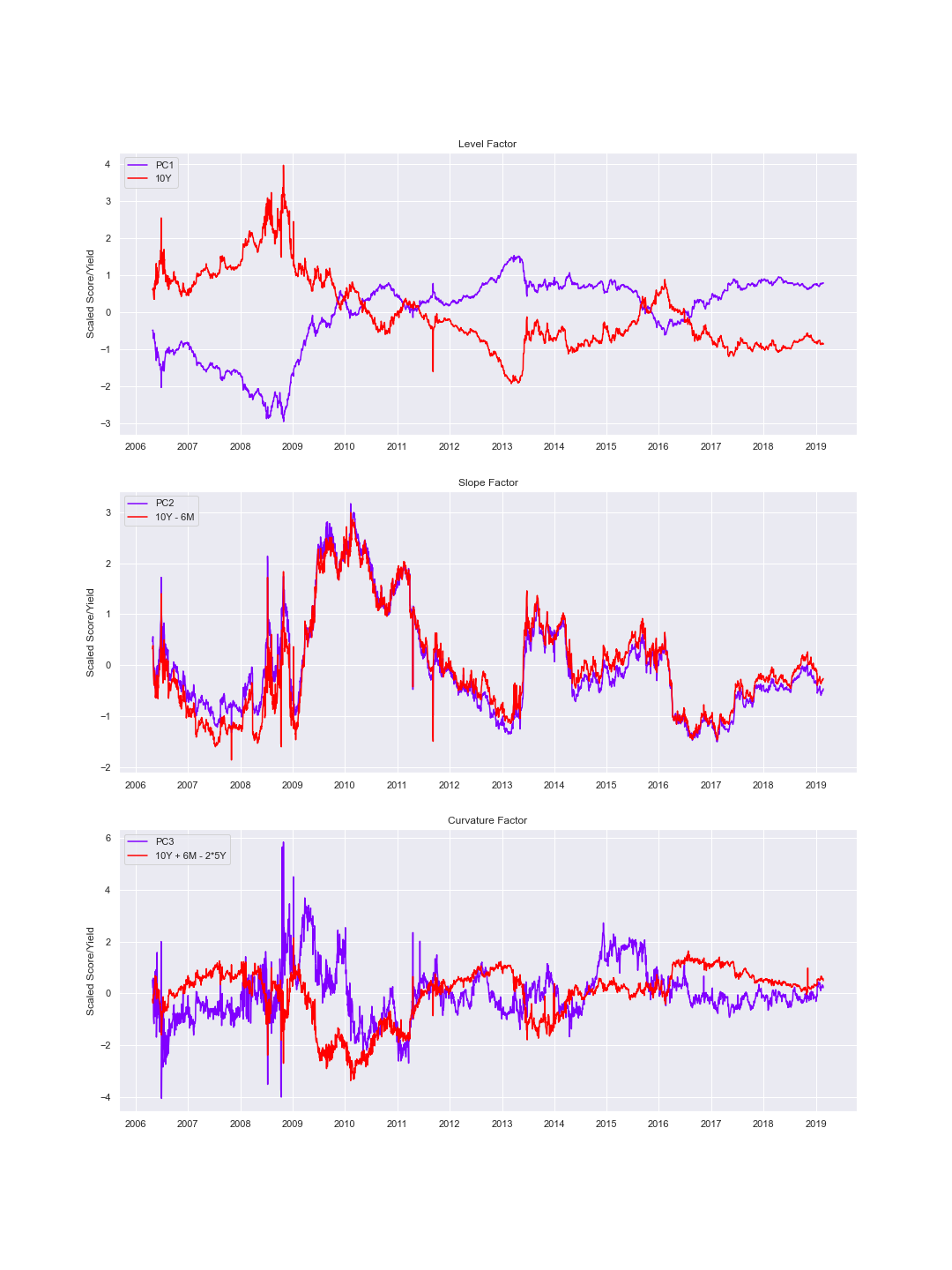
**Figure 6.** Visual representation of the PCA’s scores

Gráfico, Gráfico de dispersión

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In the literature, these factors are usually understood as a proxy of the spread between maturities, and that is why we examine the tendencies followed by the 10Y-6M spread as a proxy for steepness.

**Figure 7.** Comparation of the patrons followed by each of the three components vs. its references



Now, for the sake of the evaluation of the goodness of fit of the model, we will compare the yield curve obtained by the reverse transformation of the derived scores of the PCA process to the realized curves. The next figures present the comparison mentioned for a specific day, helping to evaluate the goodness of fit of the PCA process applied.

Gráfico, Gráfico de líneas

Descripción generada automáticamente**Figure 8.** Yield curve comparison for a specific day

Next, we also assess the predictive power out-of-sample of the underlying components by computing the metric of RMSE for the whole-time horizon. According to the figure, it exhibits very good performance.

**Figure 9.** Goodness of fit in sample and out-of-sample of the PCA method

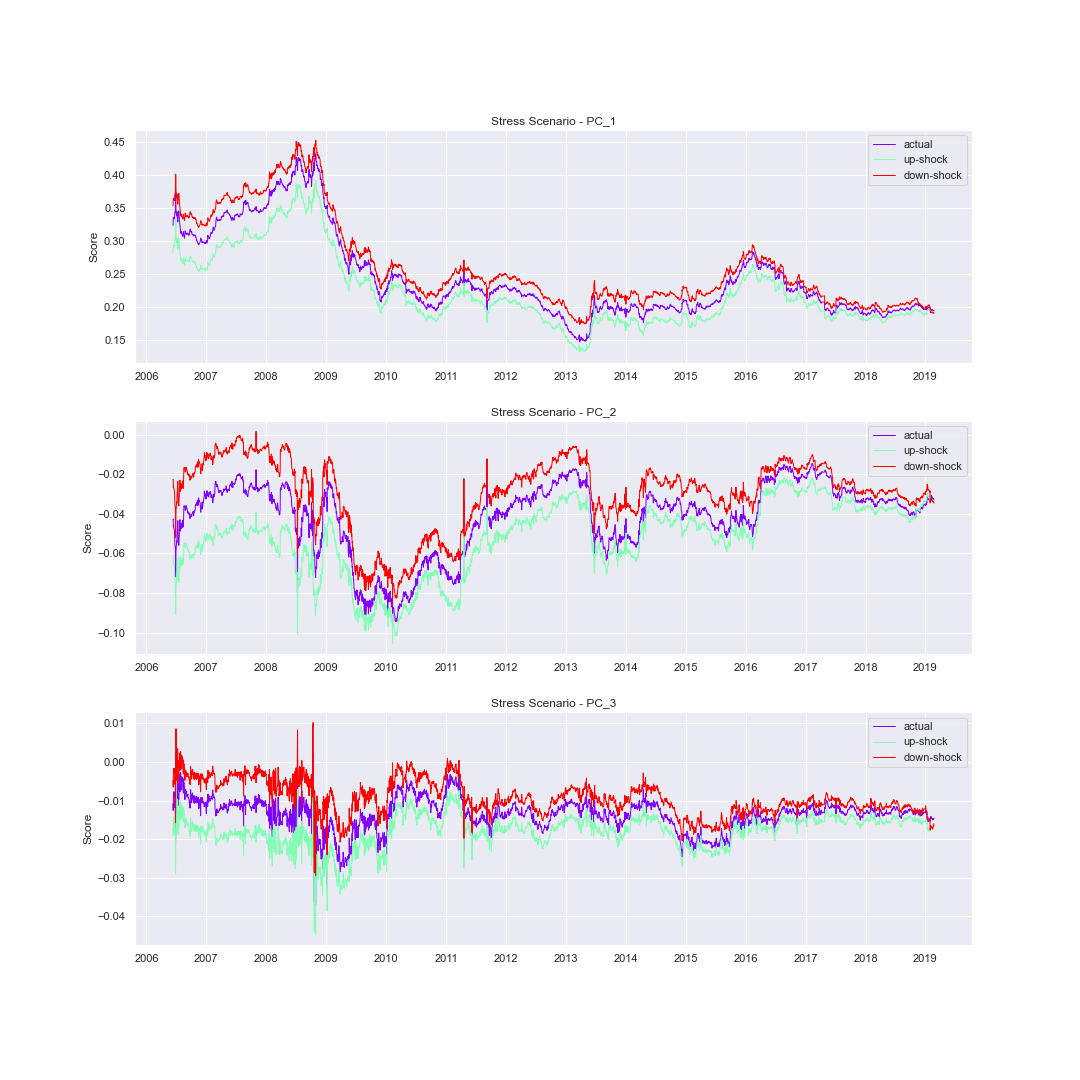
Gráfico, Histograma

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**Testing in stressing scenarios**

So far, we have demonstrated the three components extracted from the spot sovereign yield curve seize its variability solidly and consistently. Nevertheless, it is also necessary to test this technique in realistic non-linear stress scenarios. As is often done in financial analysis, we can generate parallel upward and downward shocks by applying the concept of historical Value at Risk. Specifically, we will construct a 95% confidence interval considering the 5% largest deviations within a rolling time window.

**Figure 10.** Behavior of each of the components in stress scenarios



The pattern followed by the actual yield curve never crosses out of the confidence interval of 95% for each case (PC1, PC2, and PC3), indicating a consistency in the economic interpretation of each of the factors derived for stressed scenarios according to a VaR of 5%.

**Figure 11.** Behavior of the yield curve reconstructed from the main components in stress scenarios.

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**Regression models for prediction**

As the task of prediction is not only our main concern in this work but also interpretation (considering the interest on formulate trading strategies), we propose to apply for the sake of comparing different types of models from parametric such as Naïve, AR, ARIMA to more complex and non-parametric like Decision trees and Random Forest models. It is important to note that in all of these cases, we take as an input the three principal component scores, which grant us, as we demonstrated in the last section, to fit our models with filtered and informative data.

In the following sections, we present each of the notice models, its specification, their results, and the metrics of the goodness of fit. Finally, we compare the performance of all these models using the test Diebold-Mariano, to establish if there is a statistical difference between them.

Before the stages of modeling and prediction, we also present the process of time series analysis to investigate the presence of the most common time series components, which could prevent them to be stationary in the weak sense. In that regard, we check visually for tendencies, seasonalities, cycles, and irregularities, and after that, we utilize the Augmented Dickey fuller test to verify the stationarity for each of the series analyzed.

**Stationarity analysis of the series**

As we stated before, to perform our tasks of modeling and prediction for time series data these must be stationary in mean, variance, and covariance. In that sense, we visualize the patterns of the PCA scores and afterward we check for stationarity as can be seen in the following figures:

**Figure 12.** Visualization of the time series for each of the components

Gráfico, Histograma

Descripción generada automáticamente

Stationarity-Test: PC\_1

{'adf\_stat': -1.549, 'p\_val': 0.5092, 'threshold': -2.8624, 'stationary': 'no'}

Stationarity-Test: PC\_2

{'adf\_stat': -2.556, 'p\_val': 0.1024, 'threshold': -2.8624, 'stationary': 'no'}

Stationarity-Test: PC\_3

{'adf\_stat': -3.5859, 'p\_val': 0.006, 'threshold': -2.8624, 'stationary': 'yes'}

According to the pattern presented in the graph, it can be concluded that the first component presents more marked trends and irregularities than the rest of the components. On the other hand, the third component seems, at first glance, to behave in a stationary manner in mean, variance, and autocorrelation. These observations are corroborated by the results obtained with the "Augmented Dickey-Fuller" test, for which the null hypothesis of unit roots is not rejected in the case of the first two components, while it is rejected in the case of the third component.

Considering the analysis of the results obtained, we proceed to carry out the respective transformations. Following the recommendations of the literature, we apply differences, and calculate again the Augmented Dickey-Fuller test, as can be seen below.

**Figure 13.** Visualization of the transformed time series for each of the components

Gráfico

Descripción generada automáticamente

Stationarity-Test: PC\_1\_diff

{'adf\_stat': -9.9636, 'p\_val': 0.0, 'threshold': -2.8624, 'stationary': 'yes'}

Stationarity-Test: PC\_2\_diff

{'adf\_stat': -13.1447, 'p\_val': 0.0, 'threshold': -2.8624, 'stationary': 'yes'}

Stationarity-Test: PC\_3\_diff

{'adf\_stat': -11.2339, 'p\_val': 0.0, 'threshold': -2.8624, 'stationary': 'yes'}

Both the graph and the test show that the applied transformations were effective in stationary all the series.

**AR Model**

After stationarizing the series, we proceed, first, to run the linear models AR(p) and ARIMA(p,d,q) for each of the vectors of the scores corresponding to the first three components found using the PCA method.

The general specification for AR(p) models is as follows:

Through this model, we assume that the current value of each of the series can be explained by their lags. Specifically, in our specific case, we will verify the predictive capacity of the model utilizing the first 5 lags (based on the lags indicated in the autocorrelation graphs, ACF, and the partial autocorrelation PACF). After that, we will select the best model through a more general specification such as the ARIMA model. The results obtained through the AR models can be seen below:

**Table 2.** Estimation of an AR model for each of the components

Imagen de la pantalla de un celular con letras

Descripción generada automáticamente con confianza media

Tabla

Descripción generada automáticamente

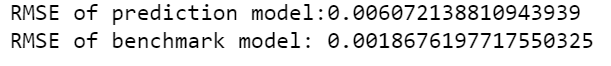
Tabla

Descripción generada automáticamente

The modeling results show a good fit in general for the three variables (the first differentiated component PC1\_diff, the second differentiated component PC2\_diff, and the third differentiated component), identifying practically all the lags proposed in each case as significant.

Now, to carry out and evaluate the prediction of the proposed models, it is necessary to calculate the inverse transformation of the scores of the components obtained (by multiplying the Eigen scores and the inverse of the eigenvectors). In such a way we obtain a matrix of the original units that represent the yield curves generated from the 3 main components.

Additionally, we compare the prediction obtained through the AR models with the prediction obtained through a model called Naive, which consists of predicting the t+1 value through the immediately previous value.



When comparing the RMSE out-of-sample values for both models, it is clear that the Naive model presents better performance. However, it is also important to establish if there is a significant difference between both models, for this we will implement the Diebold-Mariano test, not only for the mentioned models but also for the rest of the proposed models.

**ARIMA model**

Next, an ARIMA model was estimated for each of the components obtained. The selection of the model specification was carried out using the pmdarima.arima.auto\_arima package. This Python package allows an optimized search of the parameters (p,d,q), based on different criteria such as Kwiatkowski–Phillips–Schmidt–Shin, Augmented Dickey-Fuller or Phillips–Perron.

The specification and estimation results of the models implemented for components one, two and three differentiated (PC\_1\_diff, PC\_2\_diff, PC\_3\_diff) can be seen in the following tables.

The optimal specification for the data corresponding to the first component turned out to be ARIMA(3,0,4), for the second component differentiated ARIMA(2,0,1), and for the third component differentiated ARIMA(2,0,2).

**Table 3.** Estimation of an ARIMA model for the first component differentiated

Tabla

Descripción generada automáticamente

**Table 4.** Estimation of an ARIMA model for the second component differentiated

Tabla

Descripción generada automáticamente

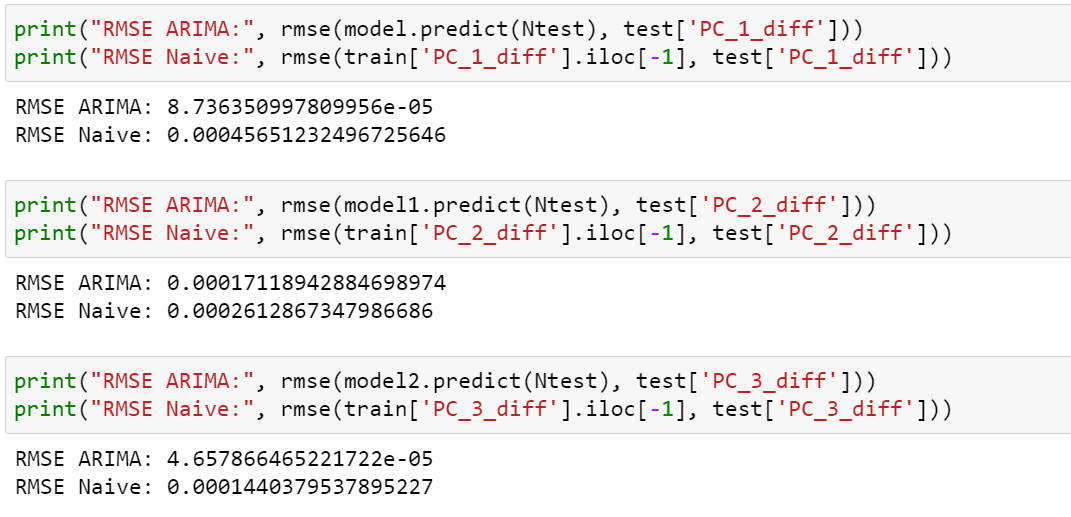
**Table 4.** Estimation of an ARIMA model for the third component differentiated

Tabla

Descripción generada automáticamente

The adjustment and comparison of the prediction for each of these models can be seen in figure 14, as well as in table 4. According to the patterns presented in the graphs, where the predicted values are very close to the real values, and with the RSME values, it is clear that the prediction capacity of the ARIMA models is much higher than that of the Naive models, confirming once plus the applicability of liquidity theory.

**Table 5.** Comparison of the root mean square error metric between the estimated models and the naive model



**Figure 14.** Adjustment level of predictions for the last five days for each of components for ARIMA models

**Gráfico, Histograma

Descripción generada automáticamente**

**Gráfico, Gráfico de líneas

Descripción generada automáticamente**

**Gráfico, Histograma

Descripción generada automáticamente**

**Random Forest Model**

Finally, in order to once again improve the prediction, trying not to overfit the model, a Random Forest was estimated. This method is characterized by using a combination of weak predictors (decision trees), sampled by bootstrap, averaging them and obtaining more accurate out-of-sample predictions than they would have using a single tree, considering their ability to reduce variance (by averaging the variance of the trees that constitute it).

Although according to the results obtained so far, everything seems to indicate that less flexible models such as linear models fit the data very well, we still wanted to experiment with non-parametric, non-linear models for comparison purposes. The results obtained can be seen in the following table.

After an exhaustive exploration of the different possibilities in terms of hyperparameters using the grid search method, the following model specification has arrived: lags =1, max\_deep= 3, and n\_estimators=100. The figures corresponding to the predictions of the last 5 days are shown below.

**Figure 15.** Adjustment level of predictions for the last five days for each of components for Random Forest models

Gráfico, Gráfico de líneas

Descripción generada automáticamente

Gráfico, Gráfico de líneas

Descripción generada automáticamente

Gráfico, Gráfico de líneas

Descripción generada automáticamente

The results show a decrease in the quality of the predictions, perhaps due to overfitting of the model. These results can be corroborated by the goodness-of-fit metrics, whose values for the first, second, and third components, respectively are shown below.





**Method Comparison for the regression models**

To finish the modeling and prediction stage, we calculate the Diebold-Mariano test in order to determine if there is a significant difference in terms of prediction between all the implemented methods. Additionally, we include the graphs corresponding to the prediction of the yield curves for the 5 days following the last date on which information is available.

Tabla

Descripción generada automáticamente con confianza baja

Texto

Descripción generada automáticamente con confianza baja

Imagen que contiene Texto

Descripción generada automáticamente

According to the results obtained there is no statistically significant difference between the estimated models, the null hypothesis is that there is no statistically significant difference between the predictions and the p-value located on the right side.

## Classification models

### 

### Toy example

As an example, we use a part of our data to obtain a deeper understanding of our method. To this end, we use the profits and losses for the {90,90} period ranging between 07/26/2006 and 08/02/2006 as our dependent variables (also known as labels). As is usual in decision tree applications, a simple transformation is performed. That is, a negative return is labeled with zero and a positive return is categorized with one. Finally, the zero-coupon yield curve rates for nodes 180 and 360 days are employed as the features. Table 6 represents the values for the toy example.

### Table 6. Values for the toy example

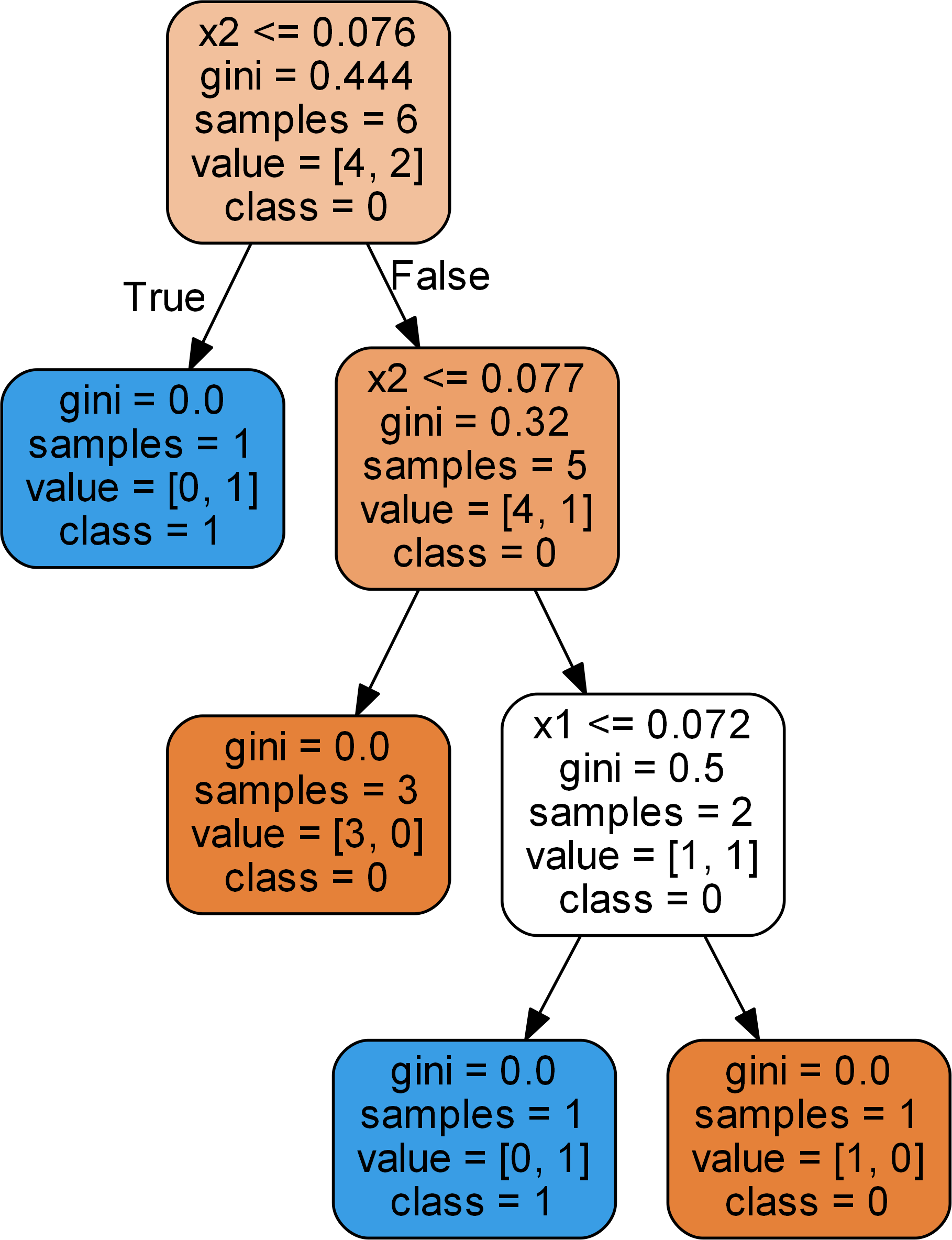
|  |  |  |  |
| --- | --- | --- | --- |
| **Return {90,90}** | **Transformation {90,90}** | **node 180** | **node 360** |
| -0.34% | 0 | 0.0733 | 0.0770 |
| -0.11% | 0 | 0.0717 | 0.0759 |
| 0.01% | 1 | 0.0712 | 0.0757 |
| -0.26% | 0 | 0.0710 | 0.0767 |
| 0.16% | 1 | 0.0710 | 0.0765 |
| 0.30% | 1 | 0.0711 | 0.0766 |

As can be inferred from Table 2, this is a linearly inseparable problem, where a simple line would not be able to separate the datapoints. Therefore, the linear regression approach or support vector machine (with linear kernel) technique would not be suitable for this type of problems. The (classification) decision tree can be used to “completely” separate these datapoints since this tool essentially draws many repeated linear limits between the datapoints.

## The Single Decision Tree (the toy model)

In the training set, the (decision) tree will learn how to separate the datapoints by “building” a diagram of questions based on the values of the features. At each stage, the decision tree is split to minimize the Gini impurity index.[[8]](#footnote-8) Usually, the bigger the tree (i.e., large number of nodes), the lesser the Gini impurity. However, a problem of data overfitting in the training set may occur when minimizing the Gini impurity index, resulting in poor modeling for the test set. In fact, an overfitting drawback can be identified using a model with “very” high accuracy rates in the training set, and low levels of accuracy in the test set. In practice, it is common to limit the depth of the tree via cross-validation or “pruning the tree” techniques. For the case of the toy example, the tree has seven nodes and a depth level of three (see Figure 1). Moreover, the accuracy is equal to 1 and the Gini impurity index is equal to zero in the last two nodes of the tree. That is, from the six observations of the sample, all of them were perfectly classified.

**Figure 16.** The single classification tree for the toy example



The five rows in each node (except the final nodes) represents the following:

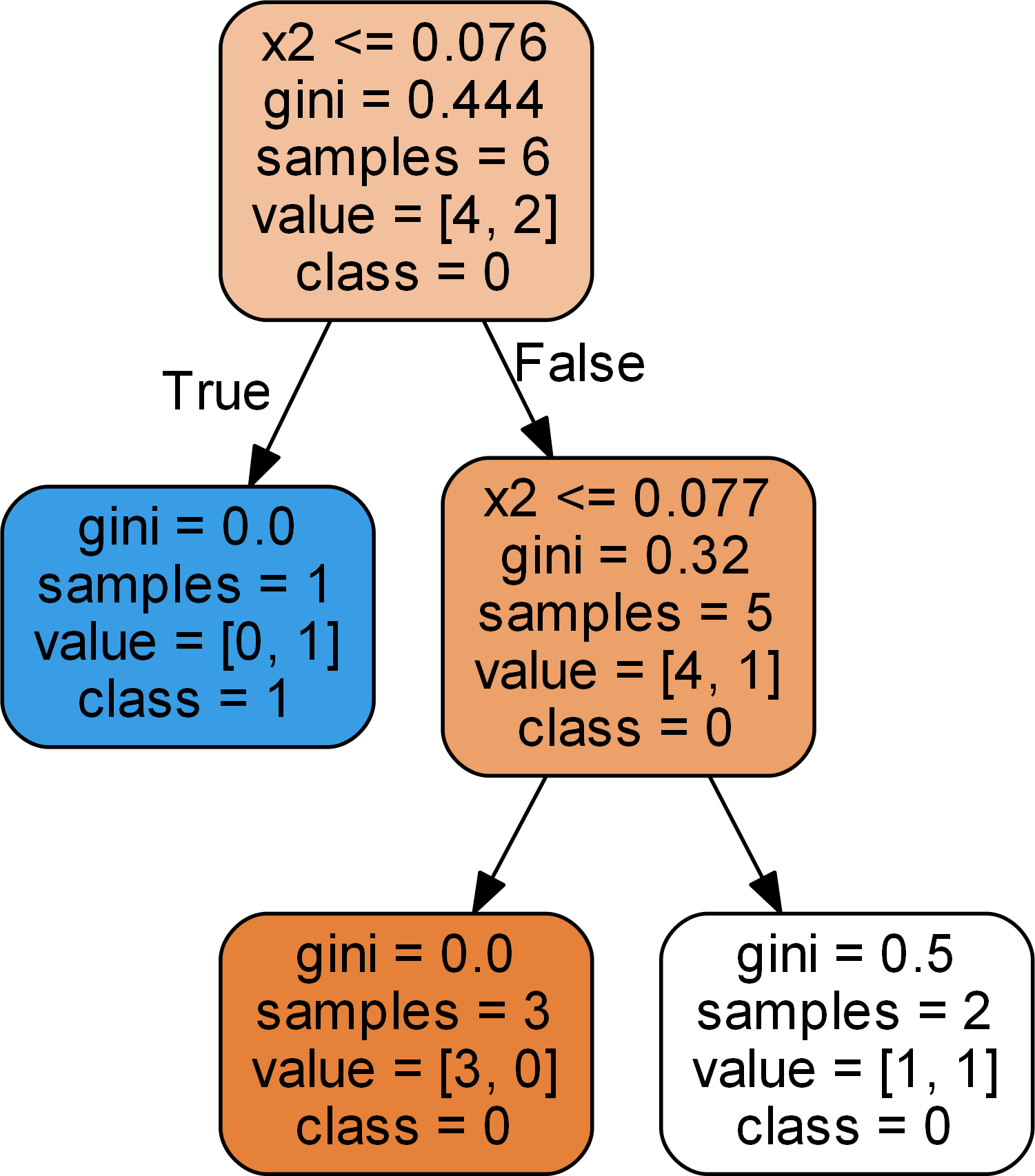
* Question asked about the features, in our case (the toy example) about the value of the interest rate in nodes 180 and 360.
* Gini: the node’s Gini Impurity index.
* Samples: the number of training observations in the node.
* Value: the number of samples in each class [0,1].
* Class: the predicted class for the entire sample of the node.

Except for the final nodes, the tree makes decisions about the class of the sample.

## Maximum Depth Limit (the toy example)

In our application, we will employ a strategy to set the size of the tree. For the toy case, we limit the maximum depth of the tree to two. In this case, the accuracy is equal to 0.67 since four of the six observations were adequately classified. Moreover, the Gini impurity index for the last (right) node is not equal to zero (it is 0.5). See Figure 2 for the results of this action.

**Figure 17.** The toy example with maximum depth equivalent to two of the classification tree



As observed in Figure 2, the tree is no more accurate in terms of the training data than it is in Figure 1, but it probably performs better on the testing set. This toy example highlights the bias − variance tradeoff, which is present in most of the machine learning models. A model with high variance learns the training data very well but it cannot perform the same with the observations on the testing set. On the other hand, a model with high bias has not learned the training data very well because of the lack of complexity, but it can generalize the results very well to the datapoints in the testing set. Our simple example shows that the bias can be improved by limiting the depth of the tree. However, another alternative is to employ an entire forest of trees. The idea is to train each tree with different subsamples of the training data and finally “averaging” each individual tree to obtain a classification or numeric prediction. In the next section, we present the results with the whole sample.

1. **Empirical Results**

This section presents the results of the machine learning techniques (decision trees and random forest) applied to the {90,90} period, which is the more balanced case for positive and negative returns. In our application, we employ three different measures: recall, precision, and AUC. The recall (also known as true positive rate) is calculated as

, (6)

where TP stands for true positives and FN the false negatives. Whereas precision is assessed as

, (7)

where FP stands for false positives. Finally, AUC is the area under the ROC curve, where ROC stands for receiver operating characteristic and it is a graph where the y-axis is the true positive rate and the x-axis is the false positive rate. Thus, the closer the ROC curve and AUC to one, the better. This is also true for the recall and precision measures.

These measures are calculated for the training set (74% of the data), test set (26% of the data), and the baseline case. The baseline case acts as the benchmark case and it represents the 45° diagonal in the ROC curve (see Figure 3), thus, the AUC for the baseline is always 0.5, and a curve above this diagonal is considered to be a good model.

* 1. **Decision tree**

This section presents the riding the yield curve strategy for period {90,90}. In the first place, we implemented the algorithm without restricting the depth of the tree (there were 299 nodes with a maximum depth of 15). Subsequently, we tuned the hyperparameters, including the maximum depth, obtaining the best model in terms of prediction. In order to validate the performance, we calculated the metrics shown in Table 3 and Figure 3.

### Table 7. Results for the strategy for period {90,90} using the decision tree algorithm optimized with cross-validation

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Baseline** | **Test** | **Train** |
| **Recall** | 1 | 0.89 | 1 |
| **Precision** | 0.6 | 0.92 | 1 |
| **AUC** | 0.5 | 0.88 | 1 |

**Figure 3.** ROC Curve for results presented in Table 3

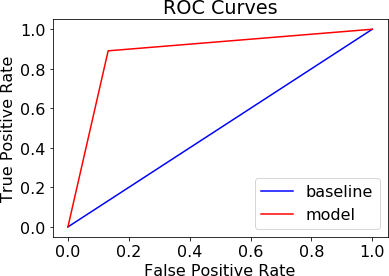
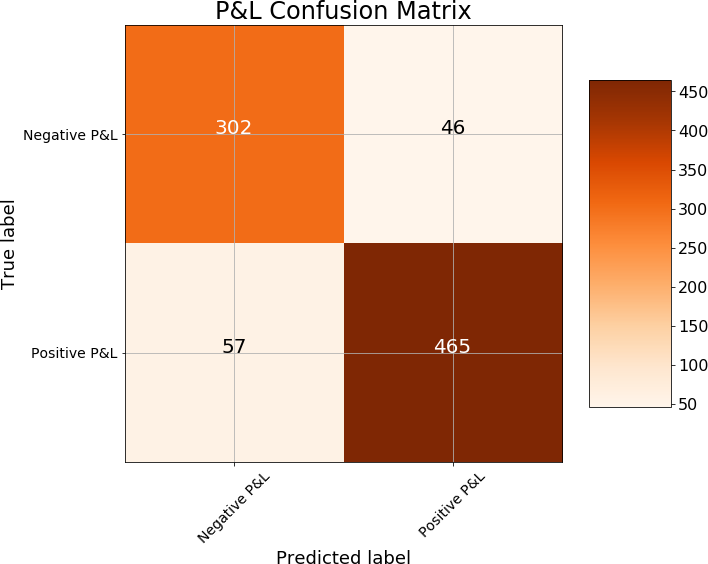


Figure 4 depicts the confusion matrix, where other measures can be calculated (namely, accuracy rate, error rate, specificity, among other indicators).

**Figure 18.** Confusion matrix for period {90,90}



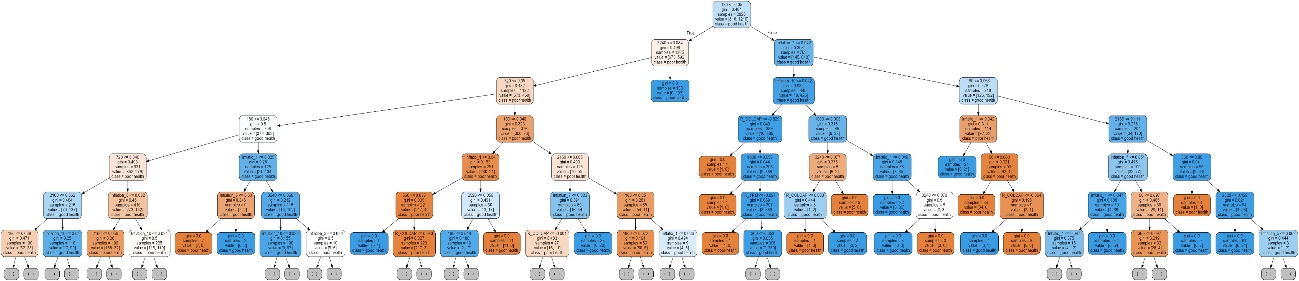
As observed in Figure 4, the number of errors in classifying observations that were positive when in fact they were negative is 46 (false negative). On the other hand, the number of errors in classifying datapoints that were negatives when in fact they were positive is 57 (false positive). Table 4 shows the most important features for the tree. The importance value is determined by the reduction in Gini impurity over all the nodes in which the feature is used.

### Table 8. Most important features for period {90,90} with classification tree

|  |  |
| --- | --- |
| **Feature** | **Importance** |
| **Node 180** | 0.289819 |
| **Node 3240** | 0.104953 |
| **1-year Inflation** | 0.093712 |
| **7-year Inflation** | 0.091384 |
| **Node 720** | 0.081656 |

The most important feature of this tree is the interest rate of node 180 in the zero-coupon yield curve since it is the variable that contributes the most to reducing Gini impurity. Figure 5 (the graph is attached as tree3.png for better visualization) depicts the classification tree for the analyzed case.

**Figure 19.** The classification tree for period {90,90}



## Random Forest

The random forest is a method that employs a certain number of individual trees. Each tree uses a random set of observations and a subset of the features are used for prediction. Table 5 and Figure 6 present the results of the random forest with 100 individual trees. With this setting, there are 274 nodes and a maximum depth of 15, on average.

### Table 9. Results of the random forest

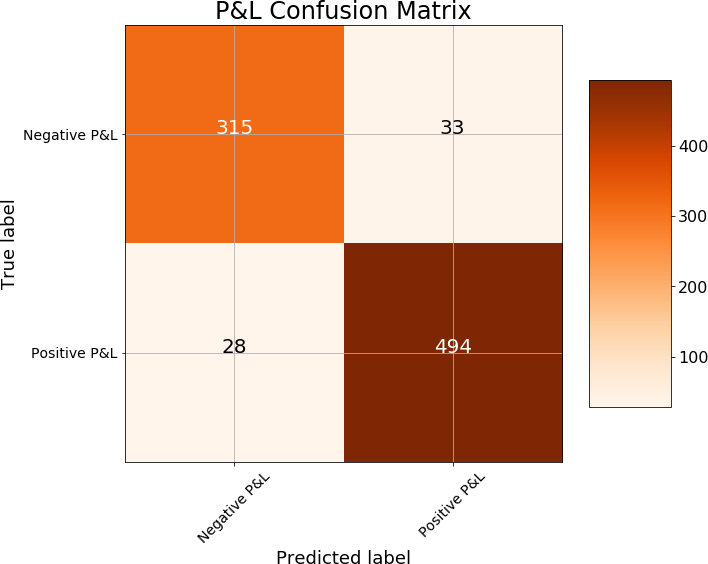
|  |  |  |  |
| --- | --- | --- | --- |
|  | **Baseline** | **Test** | **Train** |
| **Recall** | 1 | 0.95 | 1 |
| **Precision** | 0.6 | 0.94 | 1 |
| **AUC** | 0.5 | 0.98 | 1 |

**Figure 20.** ROC Curve for period {90,90} with 100 random trees



The measures for the test set in the random forest case are 0.95, 0.94 and 0.98 for precision, recall, and AUC, respectively. As observed, these results are better than those of previous case (the decision tree) for the strategy for period {90,90}. It is important to note that we selected the {90,90} period strategy considering that this period presents the more balanced classes of negative and positive returns. Figure 7 presents the confusion matrix for the analyzed period.

**Figure 21.** Confusion matrix for period {90,90}



Comparing the cases of misclassification reveals that the random forest does a better job with respect to the (individual) classification tree. The false negative is now 33 cases (before it was 46), and the false positive is 28 cases (previously it was 57). Though there are several changes in the importance feature, node 180 is still the most important variable as shown in Table 6.

### Table 10. The most important features for period {90,90} with random forest

|  |  |
| --- | --- |
| **Feature** | **Importance** |
| **Node 180** | 0.116424 |
| **Node 360** | 0.088857 |
| **Node 90** | 0.077711 |
| **Node 720** | 0.074946 |
| **Node 3240** | 0.066314 |
| **Node 2880** | 0.056533 |
| **Node 2160** | 0.052062 |
| **10-year Inflation** | 0.051956 |
| **Node 1800** | 0.051123 |
| **5-year Inflation** | 0.045169 |

Since more nodes of the zero-coupon yield curve are incorporated in the feature importance indicator in the random forest technique, this methodology employs the information in a more efficient way than the previous method (single classification tree).

## Random Forest Optimization through Random Search

To improve the performance of the random forest, we performed a random search for better hyperparameter tuning. We then evaluated the random selection of hyperparameter combinations by using cross validation of training data, and then chose the best performing parameter values for the prediction procedure.

* **bootstrap:** whether bootstrap samples are used when building trees.
* **n estimators:** the number of trees in the forest.
* **min sample split:** the minimum number of samples required to split an internal node.
* **max leaf nodes:** limits the number of leaf nodes as long as there is no improvement of impurity.
* **min samples split:** the minimum number of samples required to split an internal node.

For our application, the results of the best parameters are presented in Table 7.

### Table 11. Results of the hyperparameters after random search

|  |  |
| --- | --- |
|  | Best Parameters |
| Bootstrap | 16 |
| N estimators | 100 |
| Min sample split | 43 |
| Max leaf nodes | 5 |
| Min sample splits | 29 |

With these parameters for the random forest model, the mean of nodes is 274 and the maximum depth is 15 on average. Table 8 and Figure 8 present the results of the performance measures for the best model.

**Table 12.** Best random forest model according to random search for period {90,90}

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Baseline** | **Test** | **Train** |
| **Recall** | 1 | 0.95 | 0.98 |
| **Precision** | 0.81 | 0.94 | 0.87 |
| **AUC** | 0.50 | 0.98 | 0.88 |

**Figure 22.** ROC Curve for period {90,90} with the best random forest model



Though this model exhibits worse indicators than the previous case for the training set, the performance for the test data is the same as the previous model. In the next section, we present the results for all the cases: periods {90,90}, {180,180}, {360,360}, {720,360}, {1080, 360},{1440,360}, {1800, 360}, and {2160, 360} using the best random forest model with the hyperparameters tuned according to the random search procedure.

## Using Random Forest Optimization

For each period, we implemented the machine learning technique based on the best random forest model in riding the Colombian yield curve. We found the best hyperparameters by random search in the cross-validation procedure as previously explained. Table 9 shows the performance measures for each period in the baseline case, train set, and test set.

### Table 13. Random forest model results for different period strategies

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Baseline** | **Test** | **Train** | **Baseline** | **Test** | **Train** |
|  |  | **90, 90** |  | **180,180** | | |
| **Recall** | 1 | 0.95 | 0.98 | 1 | 0.96 | 0.99 |
| **Precision** | 0.81 | 0.94 | 0.87 | 0.72 | 0.72 | 0.99 |
| **ROC** | 0.50 | 0.98 | 0.88 | 0.50 | 0.96 | 1 |
|  |  | **360, 360** |  |  | **720, 360** |  |
| **Recall** | 1 | 1 | 1 | 1 | 0.99 | 1 |
| **Precision** | 0.86 | 0.99 | 1 | 0.85 | 0.98 | 0.99 |
| **ROC** | 0.50 | 1 | 1 | 0.5 | 1 | 1 |
|  |  | **1080, 360** |  |  | **1440, 360** |  |
| **Recall** | 1 | 1 | 1 | 1 | 0.99 | 1 |
| **Precision** | 0.93 | 0.99 | 1 | 0.98 | 1 | 1 |
| **ROC** | 0.50 | 1 | 1 | 0.5 | 1 | 1 |
|  |  | **1800, 360** |  |  | **2160, 360** |  |
| **Recall** | 1 | 0.94 | 1 | 1 | 0.96 | 1 |
| **Precision** | 0.66 | 0.95 | 0.99 | 0.25 | 0.94 | 1 |
| **ROC** | 0.50 | 0.98 | 1 | 0.5 | 1 | 1 |

As observed in Table 9, the model performs well in terms of classifying the negative and positive returns of the strategies for the test set. The precision measure is lower than 0.95 for only three cases (not shown in Table 9): periods {90,90}, {180,180}, and {2160,360}. In particular, the precision is 0.72 for the period {180, 180}, which is the same result as for the baseline case.

### Table 14. Sample sizes and number of positive and negative returns for different periods’ strategies

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **90 – 90** | **180- 180** | **360 - 360** | **720 - 360** | **1080 - 360** | **1440 - 360** | **1800 - 360** | **2160 - 360** | **2520 - 360** | **2880 - 360** | **3240 - 360** |
| ***r <* 0** | 1484 | 1115 | 692 | 523 | 147 | 30 | 568 | 1058 | 0 | 1 | 1 |
| ***r >* 0** | 1859 | 2163 | 2455 | 2363 | 2478 | 2333 | 1535 | 785 | 1582 | 1319 | 1058 |
| **N** | 3343 | 3278 | 3147 | 2886 | 2625 | 2363 | 2103 | 1843 | 1582 | 1320 | 1059 |

As we can see in Table 10, for the periods in which we do not adjust the classification model there are no or very few negative results. Thus, the problem is no longer a classification problem but one of determining the value of the profit of the strategy, which is why a different method must be applied.

**Discussion and Conclusions**

The term structure of interest rates, represented graphically by the yield curve, has been considered a powerful instrument for predicting financial crises, and the behavior of other important macroeconomic variables (economic activity, inflation, and fiscal and monetary policies, among others). In this sense, it has been and is the focus of theoretical discussions about the factors that alter its form, among which stand out the approaches of the theory of pure expectations, the theory of market segmentation, and that on which it is based the development of this work, liquidity preference.

According to this last theory, investors, in general, require a risk premium to be encouraged to buy financial assets that are valid in the medium and long term. Therefore, financial assets with such maturity periods usually present higher rates of return than those whose maturity period is shorter, hence their "normal" form, in most cases. In this research, we show that this characteristic can be used by the different type of market agents, since it is possible to predict, with a certain degree of precision, and under normal conditions, the movements of the curve.

In this sense, Machine learning techniques have been successfully applied in several financial applications. Since the work of Kondratyev (2018), there has been an increasing interest in the analysis of machine learning in the interest rate field. Our work contributes to this strand of the finance literature with the application of Machine Learning techniques to the so-called ‘riding the yield curve’ strategy.

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Our empirical evidence shows that classification trees perform well in classifying the negative and positive returns of the strategies in Colombian government bonds for the 2006 – 2019 period. We also found that regression trees present good results for the analyzed period, and that, according to the MSE values. The same as Pelaez (1997), our results show evidence in favor of the liquidity preference theory.

Regarding prediction process using regression methods, we implemented dimensionality reduction methods, decomposing the curve into the factors or components that explain its variation to a greater degree. It is noteworthy that the components obtained through the principal component analysis (PCA) coincide with the factors underlying the shape of the level curve, slope, and curvature, proposed by the theory, and that in the Colombian case they manage to explain more than 99% of the variability of the data. This last fact helps to explain the high predictive capacity of the proposed linear models.

Considering the valuable source of information that the yield curve represents for all market agents, before the modeling process, an exhaustive process of exploration and analysis of the movements of the curve was carried out for the study period (7/26/ 2006 to 2/22/20019). In general, changes in the level and steepening of the curves were observed as time progressed, and different stress scenarios were generated, using VaR as a risk measure, to verify the capacity of the main components. to explain the behavior of the curve, even in extreme cases.

Finally, the modeling and prediction process was given, in which different types of models were tested, from those characterized by less flexibility, and linearity, but greater interpretability to those that are much more flexible and non-linear, but with restrictions in terms of their performance. interpretability. According to the results obtained, linear models are superior of non-linear models in terms of prediction for the case of regression models

Future research could be very practical for the different type of market agents to improve the models in their explanatory and causal capacity. It is also suggested to explore the management of risk and the construction of portfolios taking into account different stress scenarios.

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5. National government debt bonds (TES) are used to finance the nation’s activities and the temporary management of treasury operations. Over time, they have become the most representative and widely used financial assets in the Colombian market, leveraging the capital markets and savings. [↑](#footnote-ref-5)
6. A strategy stochastically dominates another one, if the cumulative distribution function (cdf) of its returns is strictly to the right of the other. [↑](#footnote-ref-6)
7. The rate in the period for a period is . Assuming that the rates are continuous compounding, the forward rate is , where, the rate at time for a period is , and the rate at time for a period is .

   [↑](#footnote-ref-7)
8. The Gini Impurity index is the probability that a randomly selected sample from the node will be incorrectly classified. The idea is to reduce the Gini impurity, which will eventually reach 0 if the tree does not have a depth limit. The entropy index is also employed, but results do not vary substantially. [↑](#footnote-ref-8)